Please complete with your answers and let me know if you want me to review/check any of the answers.

**Modeling Problem 1**

<The following are sample modeling problems. The exam modeling problem can come from any of the chapters covered.>

A shoe manufacturer has one production plant in Pontiac and is considering opening a second plant in either Daytona or Atlanta. Each plant has a fixed cost, a variable production cost per pair and a transportation cost to the existing distribution center.

|  |
| --- |
| To Dist Center | Pontiac | Daytona | Atlanta | Demand |
| Cleveland | $ 0.42 | $ 0.44 | $ 0.48 | 10000 |
| New York | $ 0.36 | $ 0.30 | $ 0.45 | 15000 |
| San Francisco | $ 0.50 | $ 0.45 | $ 0.27 | 12000 |
| Capacity | 32000 | 40000 | 40000 |  |
| Cost/pair | $ 2.70 | $ 2.69 | $ 2.62 |  |
| Fixed Cost | $ 7,000.00 | $ 6,000.00 | $ 7,000.00 |  |

Provide the complete formulation to find the optimal location to open considering the total cost. (do not solve).

* Decision Variables:
  + d = Whether to open Daytona or not
  + a = Whether to open Atlanta or not
  + Xij = # of pairs transported from i production plant to j distribution center
    - Definitions: C = Cleveland, N = New York, S = San Francisco, P = Pontiac, D = Daytona, A = Atlanta
    - Example: XPC = Number of pairs transported from Pontiac to Cleveland
* Objective Function:
  + Minimize Total Cost (fixed + variable cost):
    - 7000+ 6000d + 7000a + (0.42+2.70)XPC + (0.36+2.70)XPN + (0.50+2.70)XPS + d(0.44+2.69)XDC + d(0.30+2.69)XDN + d(0.45+2.69)XDS + a(0.48+2.62)XAC + a(0.45+2.62)XAN + a(0.27+2.62)XAS
* Constraints (subject to):
  + d and a = binary
  + d + a =1
  + Xij = integer
  + Supply Constraints:
    - XPC + XPN + XPS <= 32000
    - XDC + XDN + XDS <= 40000d (capacity is only available if the plant is opened)
    - XAC + XAN + XAS <= 40000a (capacity is only available if the plant is opened)
  + Demand Constraints:
    - XPC + XDC + XAC >= 10000 (each location needs to receive at least their demand)
    - XPN + XDN + XAN >= 15000
    - XPS + XDS + XAS >= 12000

**Modeling Problem 2**

<The following are sample modeling problems. The exam modeling problem can come from any of the chapters covered.>

A car manufacture wants to determine the pricing of two of its cars: SUV and Wagon in a price-sensitive market, the demand for SUVs and station wagons behaves as follows:

**SUV demand = 300 – 0.014x(SUV price) + 0.003x(wagon price)**

**Wagon demand = 325 – 0.018x(wagon price) + 0.005x(SUV price)**

The SUVs/Wagons’ cost is $17K/$14K respectively and they require 2 and 3 hours of prep labor (there are 320 hours available per week). What are the profit-maximizing prices?

Provide the complete formulation to find the optimal location to open considering the total cost. (do not solve).

* Decision Variables
  + Ps = Price of SUV
  + PW = Price of Wagon
* Objective Function
  + Maximize Profit
    - (300 – 0.014PS + 0.003PW)PS + (325 – 0.018PW + 0.005PS)PW - 17000(300 – 0.014PS + 0.003PW) - 14000(325 – 0.018PW + 0.005PS)
* Constraints (Subject to)
  + 2(300 – 0.014PS + 0.003PW) + 3(325 – 0.018PW + 0.005PS) <= 320
  + Ps >= 0
  + PW >= 0

**Modeling Problem 3**

Vijay Bashwani is organizing a charity golf tournament where teams of four players will play in a captain’s choice format. The handicaps of the 40 players who have registered for the tournament are summarized in the following table. Vijay needs to create 10 teams of four players each in such a way that the total handicap of each team is as equal as possible. He would like to do this by minimizing the variance of the total handicaps of all the teams.

| Player Handicaps | |  |  |
| --- | --- | --- | --- |
| 0 | 3 | 6 | 9 |
| 0 | 3 | 6 | 9 |
| 0 | 3 | 6 | 10 |
| 0 | 4 | 6 | 10 |
| 0 | 4 | 7 | 11 |
| 1 | 4 | 7 | 11 |
| 1 | 4 | 7 | 11 |
| 1 | 5 | 8 | 12 |
| 2 | 5 | 8 | 13 |
| 2 | 5 | 8 | 13 |

Provide a model for this problem and indicate the solution strategy to use.

* Decision Variables:
  + Xi: team assignment for player i
* Objective Function:
  + Minimize variance of the total handicaps of all the teams, VAR (T1, T2, T3,..T10)

Where Ti the total handicaps of team i calculated using SUMIF

* Constraints:
  + Each team needs four player calculated using COUNTIF
  + 1<= Xi <=10

**Simulation**

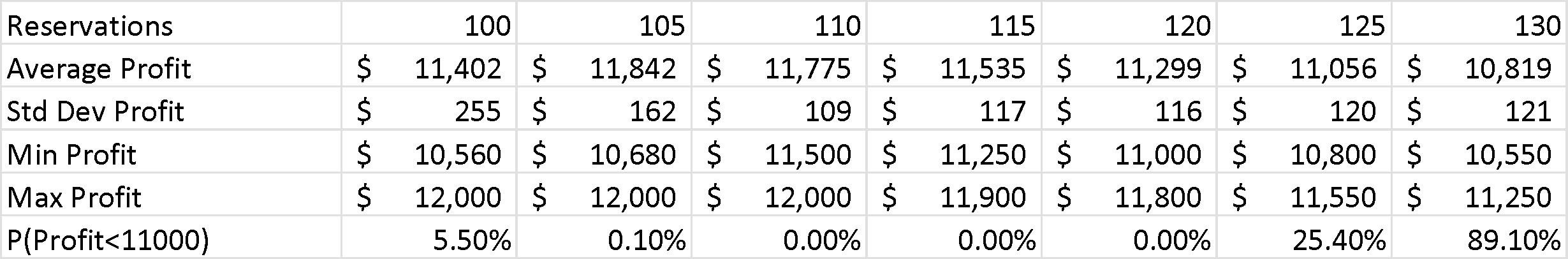
The Harriet Hotel in downtown Boston has 100 rooms that rent for $150 per night. It costs the hotel $30 per room in variable costs (cleaning, bathroom items, etc.) each night a room is occupied. For each reservation accepted, there is a 5% chance that the guest will not arrive. If the hotel overbooks, it costs $200 to relocate guests whose reservations cannot be honored to host them at the partner hotel. How many reservations should the hotel accept if it wants to maximize the average daily profit? Please assume that $150 is collected only from guests that arrive for their reservations.

1. Determine the inputs and outputs for this problem:
   1. What is(are) the input random variable(s)? Guests arriving
   2. What is(are) the decision variable(s)? Daily reservations
   3. What is(are) the output? Average daily profit
2. Determine how to calculate the output from the input information and decision variables. Clearly identify the random variable(s) and how to calculate them and all the necessary intermediate steps and calculations to reach the model output.
3. ~~The graph below shows the distribution of profit when one hundred trials were conducted assuming that 110 reservations were accepted. Explain the reasons behind the distribution of profit.~~

Chart, histogram

Description automatically generated

1. How many reservations should the hotel accept? Below are the results from a simulation considering different reservation levels.
   1. If the decision maker is risk-neutral, how many reservations? Explain your answer.
   2. If the decision maker is risk-averse, how many reservations? Explain your answer.



**Short Answer Questions:**

<Sample Questions. Expect different questions on the exam>

1. Compare the shortest path problem to the maximal flow problem. Provide similarities and differences.
   1. In both types of problems, there is only one supply node and one supply demand node. In the shortest path problem, there is only one supply/demand unit (i.e. a person traveling). For maximal flow problems, you have several supply/demand units.
2. What are the key properties of linear programming problems?
   1. Proportionality, Additivity, Divisibility, Certainty
3. When does a linear program have multiple optimal solutions? How can you recognize the existence of multiple optimal solutions from the Solver Reports?
   1. When the objective function lines up with the border of the feasible region. You can recognize this through the Solver Reports by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. What would happen to the optimal solution and objective function if the right-hand side of a binding constraint with a positive shadow price increases beyond the allowable increase?
   1. Some values in the optimal solution will increase and others will decrease. The objective function\_\_\_\_(help?)
5. Compare and contrast linear programming to nonlinear programming.
   1. Linear programming always gives a global optimal solution, while nonlinear programming can give either a local or global optimal solution, and this depends on the initial input values for the decision variables.
   2. Both are examples of constrained optimization models
   3. Non-linear programming uses GRG nonlinear while linear programming uses Simplex LP
   4. The objective function and relationships between decision variables in non-linear programming can take on non-linear forms (multiplication, exponents, etc). There can also be one or more non-linear constraints.
6. How does branch and bound work?
   1. Branch and bound is mainly an integer programming, you first solve the problem without integer constraints. If the answer is an integer then we can keep it. If not,. For example,Type A fibers = 116.94 If it’s not an integer then we have to solve the two new problems: one with constraint can have type A fibers that is <= 116 and the other with number of type A fibers is >=117. We use the term branch here as it parts it and bound for the lower bound of 116 and upper for 117.
7. What are some similarities and differences between tabu search and genetic algorithms?
   1. The tabu search algorithm is a local search procedure that uses the steepest-ascent mildest-descent approach to find a solution. It’s shortcoming is that it may be unable to find a global optimal solution and get stuck on a local optimum. For example, after moving away from a local optimum, it will cycle back towards that optimum. The genetic algorithm, on the other hand, is concerned with populations of trial solutions and generates improving populations of trial solutions as it proceeds. It uses a random process that is biased towards certain more fit members of the current population.

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**Multiple Choice Questions:**

<Sample questions. A great source of additional questions are the chapter quizzes>

1. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution to an integer programming problem.
   1. TRUE
2. The appropriate criterion is dependent on the risk personality and philosophy of the decision maker.
   1. TRUE
3. An assignment problem is a special form of transportation problem where all supply and demand values equal 1.
   1. TRUE
4. Sensitivity analysis can be used to determine the effect on the solution for changing several parameters at once.
   1. FALSE
5. A change in the value of an objective function coefficient will always change the value of the optimal solution.
   1. FALSE
6. The sensitivity range for a constraint quantity value is the range over which the shadow price is valid.
   1. TRUE
7. The flaws of averages indicates that
   1. The average is not always a good description of the actual situation,
   2. The function of the average is not always the same as the average of the function,
   3. The average depends on your perspective.
   4. **All of the above,**
   5. None of the above.
8. A rounded-down integer solution CANNOT result in a less than optimal solution to an integer programming problem.
   1. FALSE
9. In a \_\_\_\_\_\_\_\_\_\_ linear programming model, the solution values of the decision variables are whole numbers.
   1. INTEGER
10. In a 0-1 integer program, if one investment requires the adoption of another investment too, this would be written as: \_\_\_\_\_\_\_\_\_\_.
    1. X <= Y, WHERE X AND Y = BINARY, AND Y IS THE ADOPTION THAT IS NEEDED FOR X
11. Sensitivity analysis can be used to determine the effect on the solution for changing one parameter at a time.
    1. TRUE
12. If we change the constraint quantity to a value outside the sensitivity range for that constraint quantity, the shadow price will change.
    1. TRUE
13. The sensitivity range for an objective function coefficient is the range of values over which the profit does not change.
    1. FALSE, total profit can change but the numbers for the decision variables will not change
14. A plant manager is attempting to determine the production schedule of various products to maximize profit. Assume that a machine hour constraint is binding. If the original amount of machine hours available is 200 minutes., and the range of feasibility is from 130 minutes to 340 minutes, providing two additional machine hours will result in:
    1. the same product mix, same total profit
    2. a different product mix, different total profit
    3. **the same product mix, different total profit**
    4. a different product mix, same total profit as before
15. A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_represents a limitation to achieving maximum profits due to limited resources.
    1. CONSTRAINT
16. In Excel, the \_\_\_\_\_\_\_\_\_\_ function can be used to generate random number.
    1. RAND()
17. Developing the cumulative probability distribution helps to determine random number ranges.
    1. TRUE

**Additional Practice Problems**

**Problem 1**

Ocean City Beach is a popular vacation destination for thousands of people. Each summer, the city hires temporary lifeguards to ensure the safety of the vacationing public. Lifeguards are assigned to work five consecutive days each week and then have two days off. However, the city’s insurance company requires them to have a least the following number of lifeguards on duty each day of the week

|  | Minimum number of lifeguards required each day | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| Lifeguards | 18 | 17 | 16 | 16 | 16 | 14 | 19 |

The city would like to determine the minimum number of lifeguards that will have to be hired.

1. Formulate an ILP for this problem.
   1. Define decision variables.

Xi: number of lifeguards that start their 5 day shift on day i.

For example X\_sunday = 10, means that 10 lifeguards will be on duty for 5 days starting on Su, then they will be on duty Su, Mo, Tu, We and Th

* 1. Formulate objective function.

Minimize the number of lifeguards to hire: X\_Sunday + X\_Monday + X\_Tuesday + X\_Wednesday + X\_Thursday + X\_Friday + X\_Saturday

* 1. Determine and formulate all constraints.

Xi >= 0

Xi is an integer

>= 18

X\_Thursday + X\_Friday + X\_Saturday + X\_Sunday + X\_Monday >= 17

X\_Friday + X\_Saturday + X\_Sunday + X\_Monday + X\_Tuesday >= 16

X\_Saturday + X\_Sunday + X\_Monday + X\_Tuesday + X\_Wednesday >= 16

X\_Sunday + X\_Monday + X\_Tuesday + X\_Wednesday + X\_Thursday >= 16

X\_Monday + X\_Tuesday + X\_Wednesday + X\_Thursday + X\_Friday >= 14

X\_Tuesday + X\_Wednesday + X\_Thursday + X\_Friday + X\_Saturday >= 19

1. Several lifeguards have expressed a preference to be off on Saturdays and Sundays. Reformulate the ILP for the problem to maximize the number of lifeguards that can be off on weekends without increasing the total number of lifeguards required.

Let’s assume that after solving the model in part a the minimum number of lifeguards needed is C.

Then the new formulation would be as follows:

Max X\_Monday

Subject to

Xi >= 0

Xi is an integer

X\_Wednesday + X\_Thursday + X\_Friday + X\_Saturday + X\_Sunday >= 18

X\_Thursday + X\_Friday + X\_Saturday + X\_Sunday + X\_Monday >= 17

X\_Friday + X\_Saturday + X\_Sunday + X\_Monday + X\_Tuesday >= 16

X\_Saturday + X\_Sunday + X\_Monday + X\_Tuesday + X\_Wednesday >= 16

X\_Sunday + X\_Monday + X\_Tuesday + X\_Wednesday + X\_Thursday >= 16

X\_Monday + X\_Tuesday + X\_Wednesday + X\_Thursday + X\_Friday >= 14

X\_Tuesday + X\_Wednesday + X\_Thursday + X\_Friday + X\_Saturday >= 19

X\_Sunday + X\_Monday + X\_Tuesday + X\_Wednesday + X\_Thursday + X\_Friday + X\_Saturday <= C

**Problem 2**

Radford Castings can produce brake shoes on six different machines. The following table summarizes the manufacturing costs associated with producing the brake shoes on each machine along with the available capacity on each machine. If the company has received an order for 1,800 brake shows, how should it schedule these machines?

| Machine | Fixed Cost | Variable Cost | Capacity |
| --- | --- | --- | --- |
| 1 | $1,000 | $21 | 500 |
| 2 | $950 | $23 | 600 |
| 3 | $875 | $25 | 750 |
| 4 | $850 | $24 | 400 |
| 5 | $800 | $20 | 600 |
| 6 | $700 | $26 | 800 |

1. Formulate the ILP for this problem.
   1. Define decision variables.
      1. Whether Machine i should be turned on: Mi
      2. How many shoes a machine should produce if it is used: Xi
   2. Formulate objective function.
      1. Minimize total cost (fixed + variable cost)
      2. M1(1000 + 21X1) + M2(950 + 23X2) + M3(875 + 25X3) + M4(850 + 24X4) + M5(800 + 20X5) + M6(700 + 26X6)
   3. Determine and formulate all constraints.
      1. Mi = binary
      2. X1 <= 500
      3. X2 <= 600
      4. X3 <= 750
      5. X4 <= 400
      6. X5 <= 600
      7. X6 <= 800
      8. Xi >= 0, int
      9. Sum Xi = 1800
2. Provide the constraints for the following additional requirements:
   1. If Machine 1 is scheduled, Machine 2 should be also scheduled.
      1. M1 = M2
   2. If Machine 3 is scheduled, Machine 4 should be also scheduled but not vice versa.
      1. M3 <= M4
   3. Either machine 1 or machine 5 must be scheduled.
      1. M1 + M5 = 1

**Problem 3**

The Britts & Straggon company manufactures small engines at three different plants. From the plants, the engines are transported to two different warehouse facilities before being distributed to three wholesale distributors. The per-unit manufacturing cost at each plant is shown in the following table in addition to the minimum required and maximum available daily production capacities.

| Plant | Manufacturing Cost | Minimum Required Production | Maximum Production Capacity |
| --- | --- | --- | --- |
| 1 | $13 | 150 | 400 |
| 2 | $15 | 150 | 300 |
| 3 | $12 | 150 | 600 |

The unit cost of transporting engines from each plant to each warehouse is show in the following table:

| Plant | Warehouse 1 | Warehouse 2 |
| --- | --- | --- |
| 1 | $4 | $5 |
| 2 | $6 | $4 |
| 3 | $3 | $5 |

The unit cost of shipping engines from each warehouse to each distributor is shown in the following table along with the daily demand for each distributor.

| Warehouse | Distributor 1 | Distributor 2 | Distributor 3 |
| --- | --- | --- | --- |
| 1 | $6 | $4 | $3 |
| 2 | $3 | $5 | $2 |
| Demand | 300 | 600 | 100 |

Each warehouse can process up to 500 engines per day.

* + - 1. Formulate the LP model to determine the least costly production and transportation plan. (It is highly recommended that you draw the network to assist you in the process of formulating the problem.)
         1. Define decision variables.

Xipjc: Number of engines i transported from p 1,2,3 to warehouse j4,j5 then distributed to c6,c7,c8

* + - * 1. Formulate objective function.

Minimize Total Cost

* + - * 1. Determine and formulate all constraints.